# Lifting Properties In Skew-product Flows With Applications To Differential Equations

February 24, 2012 17:59 Journal of Biological Dynamics Sigmoid-BevHolt-v5

*Journal of Biological Dynamics*
Vol. 00, No. 00, February 2012, 1–12

**Difference Equations with the Allee Effect and the Periodic Sigmoid Beverton-Holt Equation Revisited**

Garren R. J. Gaut*¶, Katja Goldring†¶, Francesca Grogan§¶, Cymra Haskell‡‡, Robert J. Sacker††
(*Received 00 Month 200x; revised 00 Month 200x; in final form 00 Month 200x*)

In this paper we investigate the long term behavior of solutions of the periodic Sigmoid Beverton-Holt equation

$$x_{n+1} = \frac{a_n x^{\delta_n}}{1 + x^{\delta_n}}, \quad x_0 > 0, \quad n = 0, 1, 2, \ldots,$$

where the $a_n$ and $\delta_n$ are $p$-periodic positive sequences. Under certain conditions there are shown to exist an asymptotically stable $p$-periodic state and a $p$-periodic Allee state that repels nearby states. This appears to be the first study of the equation with variable $\delta$.

*Keywords*: Periodic difference equation, global stability, Sigmoid Beverton-Holt, Allee states

*AMS Subject Classification:* 39A11, 92, 92D, 92D25

## 1. Introduction

In this paper we investigate the long term behavior of solutions of the periodic Sigmoid Beverton-Holt (or Holling Type III, [8]) equation

$$x_{n+1} = \frac{a_n x^{\delta_n}}{1 + x^{\delta_n}}, \quad x_0 > 0, \quad n = 0, 1, 2, \ldots, \tag{1.1}$$

where the $a_n$ and $\delta_n$ are $p$-periodic positive sequences. In a recent groundbreaking publication by Harry, Kent and Kocic [7] an extensive study was made in the case $\delta$ =constant and a rich source of references on the subject were presented. Technically, the term "Sigmoid" applies only to the case in which $\delta > 1$ where the graph of what we call the Sigmoid Beverton-Holt function,

$$f_{a,\delta}(x) = \frac{ax^{\delta}}{1 + x^{\delta}}, \quad a > 0,$$

has the characteristic "S" shape, the slow rise from zero, a rapid rise then flattening out for large $x$. This shape is especially interesting in discrete dynamics when for '$a$' sufficiently large it gives rise to the famous Allee effect in which small populations are driven to extinction. This is of paramount importance in the management of fisheries and establishment of safeguards against overfishing [1], [12]. In [17], Stephens and Sutherland described several scenarios that cause the Allee effect in both animals and plants. For example, cod and many freshwater fish species have high juvenile mortality when there are fewer adults. Fewer red sea urchin give rise to worsening feeding conditions of their young and less protection from predation. In some mast flowering trees, such as *Spartina alterniflora*, low population density results in lower probability of pollen grains finding stigma [17]. See [4] for a discussion of some new examples of models exhibiting the

††Corresponding author. Email: rsacker@usc.edu, http://bcf.usc.edu/∼rsacker
¶ University of California, Los Angeles
* Email: ggaut@ucla.edu, † robyweah@gmail.com, § fcgrogan@yahoo.com
‡‡ Email: chaskell@usc.edu

*Journal of Biological Dynamics*
ISSN: 1751-3758 print/ISSN 1751-3766 online 
http://www.tandf.co.uk/journals
DOI: 10.1080/1751375YYxxxxxxxx

Full-Text Paper (PDF): Lifting properties in skew-product flows with applications to differential equations / by Robert J. Sacker and George R. Sell.Lifting properties in skew-product flows with applications to differential equations. Article (PDF Available) in Memoirs of the American Mathematical Society.Lifting Properties in Skew-Product Flows with Applications to Differential Equations cover image. Memoirs of the American Mathematical.Lifting Properties in Skew-Product Flows with Applications to Differential Equations. Front Cover Chapter III Applications to Ordinary Differential Equations. Lifting Properties in Skew-product Flows with Applications to Differential Equations, Issues Front Cover. Anthony Ayers Iarrobino, Michael H. Freedman.We consider a skew-product flow with a minimal flow as the basic .. V. V. Zhikov , Almost periodic functions and differential equations, Cambridge R. J. Sacker and G. R. Sell, Lifting properties in skew-product flows with applications to differ-.we show several significant applications of the abstract theory of topological of a skew-product flow from a non-autonomous ordinary differential equation: .. lifting properties in skew-product semi-flows related to qualitative study of non-.#1 Book Source: Lifting Properties In Skew Product Flows With Applications To Differential Equations Memoirs Of The American Mathematical.be the phase space for the differential equation, usually X is the Euclidean . Lifting properties in skew-product flows with applications to differential equations.culus by Isaac Newton in and the dominance of differential equations [ 63] Sacker, R.J. and G.R. Sell, Lifting properties in skew-product flows with.Keywords: Difference equation; Population biology; Skew-product dynamical .. G. Sell, Lifting properties in skew-product flows with applications to differential.R. Sacker and G. Sell, Lifting properties in skew-product flows with applications to differential equations, Memoirs of the Amer. Math. Soc. No. Exponential dichotomy and trichotomy for difference equations. Comp. Math J. Differential Equations, (), pp. Lifting Properties in Skew- Product Flows with Applications to Differential Equations. American.Skew-product flows. Singular problems for non linear differential equations; singular perturbation . Lifting properties In skew-product fl ows with applications.Differential equations and dynamical systems (Proc. "Lifting properties in skew- product flows with applications to Differential Equations," (with G. Sell).In this note, we study the k-periodic Ricker equation for . equations. Journal of Differential Equations, (11): January Lifting properties in skew- product flows with applications to differential equations. Memoirs.Sacker R J and Sell G R Lifting Properties in Skew-product Flows with Applications to Differential Equations (Memoirs of the American Mathematical.Lifting Properties in Skew-product Flows With Applications to Differential Equations (Memoirs of the American Mathematical Society). No Image Available. $R J Sacker and G R Sell (), Lifting Properties in Skew Product Flows with Applications to Differential Equations, Memoir Amer Math Soc, No , ferential equations. cal method of detecting bounded solutions of some nonautonomous differential Nonautonomous equation, bounded solution, flow, process, .. Sell G.R., Lifting properties in skew-product

flows with applications to dif-.The paper deals with order-preserving (or monotone) skew-product flows in Banach concavity property called sublinearity, frequently encountered in applications. Our main examples are quasilinear systems of parabolic equations with Sell, G. R.: Lifting properties in skew-product flows with applications to differential.